

Rectangular cavity has described by single scalar function A_x (x component of the magnetic vector potential). The wave equation becomes,

$$\Delta^2 A_x + K^2 A_x = 0$$

$$A_x = X(x) Y(y) Z(z)$$

The Eigen value equation is,

$$K_x^2 + K_y^2 + K_z^2 = K^2$$

Here K_x , K_y , K_z are the wave numbers along the x,y,z directions respectively. The electric field components inside the cavity are related to vector potential A_x .

$$E_x = -J \frac{1}{w\mu\epsilon} \left(\frac{\partial^2}{\partial x^2} + K^2 \right) A_x$$

$$E_y = -J \frac{1}{w\mu\epsilon} \left(\frac{\partial^2 A_x}{\partial x \partial y} \right)$$

$$E_z = -J \frac{1}{w\mu\epsilon} \left(\frac{\partial^2 A_x}{\partial x \partial z} \right)$$

$$H_x = 0$$

$$H_y = \frac{1}{\mu} \left(\frac{\partial A_x}{\partial z} \right)$$

$$H_z = -\frac{1}{\mu} \left(\frac{\partial A_x}{\partial y} \right)$$

Applying harmonic functions in the equation (3.3) should derive the general solutions. That particular solutions depends on the boundary conditions. The boundary conditions are applied as per the x, y, and z coordinates where cavity is placed.

$$E_y(x' = 0, 0 \leq y' \leq L, 0 \leq z' \leq W) = E_y(x' = h, 0 \leq y' \leq L, 0 \leq z' \leq W) = 0$$

$$H_y(0 \leq x' \leq h, 0 \leq y' \leq L, z' = 0) = H_y(0 \leq x' \leq h, 0 \leq y' \leq L, z' = W) = 0$$

$$H_z(0 \leq x' \leq h, y' = 0, 0 \leq z' \leq W) = H_z(0 \leq x' \leq h, y' = L, 0 \leq z' \leq W) = 0$$

From this boundary conditions, we can simply derive the wave numbers,

$$K_x = \left(\frac{m\pi}{h} \right) \quad m = 0, 1, 2 \dots$$

$$K_y = \left(\frac{p\pi}{W} \right) \quad p = 0, 1, 2 \dots$$

$$K_z = \left(\frac{n\pi}{L}\right) \quad n = 0, 1, 2, \dots$$

Here $[m = n = p \neq 0]$. The equation (3.5) becomes,

$$K_x^2 + K_y^2 + K_z^2 = \left(\frac{m\pi}{h}\right)^2 + \left(\frac{p\pi}{W}\right)^2 + \left(\frac{n\pi}{L}\right)^2 = K^2 = \omega^2 \mu \epsilon$$

The resonant frequencies for the cavity are given by,

$$(f_r)_{mnp} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{p\pi}{W}\right)^2 + \left(\frac{n\pi}{L}\right)^2}$$

For dominant mode the equation (3.17) should get lowest resonant frequency at TM 010.

$$(f_r)_{010} = \frac{1}{2L\sqrt{\mu\epsilon}} = \frac{v}{2L\sqrt{\epsilon r}}$$